UCRL- 94993
PREPRINT

DECENTRALIZED OPTIMAL CONTROL OF LARGE FLEXIBLE STRUCTURES

Shih-Ho Wang
Stephen C. Lu
I-Kong Fong

This paper was prepared for submittal to
the 29th Midwest Symposium on
Circuits and Systems
August 11-12, 1986

July 14, 1986

DISCLAIMER

DECENTRALIZED OPTIMAL CONTROL OF LARGE FLEXIBLE STRUCTURES

Shih-Ho Wang *
Stephen C. Lu
I-Kong Fong **

University of California
Lawrence Livermore National Laboratory
Livermore, CA 94550

ABSTRACT

This paper studies the problem of controlling large flexible structures by using decentralized feedback control. The proposed algorithm is initially applied to the control of a flexible beam. Two independent forces are applied at each tip of the beam. One displacement sensor and one velocity sensor are colocated with each force actuator. Computer simulations indicate that the decentralized feedback is effective in suppressing the structural vibrations of the beam.

* Consultant for Lawrence Livermore National Laboratory, also Professor of Electrical and Computer Engineering, University of California, Davis.

** Visiting Scientist at the Department of Electrical and Computer Engineering, University of California, Davis.

1. INTRODUCTION

This paper studies the problem of controlling large scale structural systems to meet certain stringent performance specifications. The motion of a large flexible structure is usually modelled, via finite element method, by a set of linear differential equations. However, due to the large number of degrees of freedom, the finite element models of large structures are of extremely high order (typically thousands of variables). Furthermore, in order to maintain the desired performance of a large flexible structure, large numbers of sensors and actuators are necessary.

To control such complex systems, new and innovative methods are necessary. In this paper, we propose to apply the Decentralized Optimal Control Theory [1], [2] to our problem. We have completed a design of decentralized controller for a flexible beam in order to demonstrate the proposed technique. This flexible beam problem was first studied by Gavel and Woo [3], for which a centralized optimal controller was constructed.

We started with a finite-element model of this beam with 51 modes. In our reduced-order design model, we have included one rigid body and five elastic modes. One force actuator is assumed to be located at each tip of the beam. One displacement and one velocity sensor are colocated with each force actuator. We are able to find a set of decentralized feedback gains which stabilize the system. Computer simulations show that structural vibrations are well suppressed.

2. DECENTRALIZED OPTIMAL CONTROL

In [1], Yanchevsky and Hirvonen have proposed an interesting algorithm which constructs a sequence of feedback matrices for decentralized optimal control problems. Specifically, consider the following discrete-time system

$$x_{t+1} = Ax_t + Bu_t \tag{1}$$

where $x_t \in R^n$, $u_t \in R^m$ are the state and input, A and B are constant matrices of appropriate sizes. The usual quadratic performance index is defined as

$$J = x_T' Q_T x_T + \sum_{t=o}^{T-1} (x_t' Q x_t + u_t' R_o u_t) \qquad (2)$$

where Q_T and Q are assumed to be positive semi-definite, and R_o is assumed to be positive definite.

The optimal control problem is to find a sequence of matrices K_t, such that the feedback control $u_t = - K_t x_t$ minimizes the performance index J in (2). Let $k_t^{i,j}$ be the element at the i-th row and the j-th column of the matrix K_t. Let u_t^i and x_t^i be the i-th component of u_t and x_t, respectively. Then u_t^i can be written as

$$u_t^i = u_t^{i,1} + \ldots + u_t^{i,n} = - k_t^{i,1} x_t^1 - \ldots - k_t^{i,n} x_t^n$$

$$= - \sum_{j=1}^{n} k_t^{i,j} x_t^j$$

The augmented performance index is as follows:

$$J = x_T' Q_T x_T + \sum_{t=o}^{T-1} (x_t' Q x_t + u_t' R_o u_t + \sum_{i=1}^{m} \sum_{j=1}^{n} u_t^{i,j} r_{i,j} u_t^{i,j}) \qquad (3)$$

The nonnegative numbers $r^{i,j}$ may be interpreted as the cost of feedback from the j-th component of x_t to the i-th component of u_t.

Let e^i denote an n-dimensional column vector whose i-th component is equal to 1 and others are equal to zero. Then the performance index (3) can be rewritten as

$$J = x_T' Q_T x_T + \sum_{t=o}^{T-1} x_t' (Q + K_t' R_o K_t + \sum_{i=1}^{n} e_i e_i' K_t' R_i K_t e_i e_i') x_t \qquad (4)$$

where R_i are diagonal matrices $R_i = \text{diag}\,(r_{1,i}, r_{2,i}, \ldots, r_{m,i})$, $(i=1,\ldots,n)$. Using dynamic programming technique to minimize (4), the following iterative formula is obtained,

$$K_{t-1}^i = (B' P_t B + R_o + R_i)^{-1} D_t^i, \; (i=1,\ldots,n)$$

$$P_{t-1} = Q + (A-BK_{t-1})' P_t (A-BK_{t-1}) + K'_{t-1} R_o K_{t-1}$$

$$+ \sum_{i=1}^{n} e_i e_i' K'_{t-1} R_i K_{t-1} e_i e_i' \qquad (5)$$

$$D_t = B' P_t A$$

$$P_T = Q_T$$

$$J = x_o' P_o x_o$$

where D_t^i and K_t^i , $(i=1,\ldots,n)$ are the i-th columns of matrices D_t and K_t, respectively.

Procedure (5) generates a sequence of feedback matrices K_t for the performance index in (4), and it converges to the steady state solution K if system (1) is stabilizable. We consider the case of decentralized control by setting the cost of feedback $r^{i,j}$ to either o or $+\infty$, depending on whether the state x_t^j is available for evaluating u_t^i or not. Procedure (5) can be modified as follows:

$$K_{t-1}^i = F((B' P_t B + R_o), R_i) D_t^i, \ (i=1,\ldots,n)$$

$$P_{t-1} = Q + (A-BK_{t-1})' P_t (A-BK_{t-1}) + K'_{t-1} R_o K_{t-1} \qquad (6)$$

$$D_t = B' P_t A$$

$$P_T = Q_T$$

$$J = x_o' P_o \ x_o$$

where F (·) is defined as follows:

$$F((B' P_t B + R_o), R_i) = \lim_{r^{i,j} \to \infty, \ (i,j) \in S} \ldots \lim (B' P_t B + R_o + R_i)^{-1},$$

and $S = \{ (i,j) \mid k_t^{i,j}$ is required to be zero for all $t \}$.

For more details on the above algorithm, see [1], [2].

3. DYNAMIC MODEL FOR A FLEXIBLE BEAM

To evaluate the effectiveness of the Decentralized Optimal Control Theory stated in the last section, we adopt a specific example from a paper by Gavel and Woo [3]. In this example, a one meter long flexible beam structure, as shown by Fig. 1, is to be controlled. The beam is pivoted in the middle, and has two 1 kilogram lumped masses at 30 cm and 70 cm from one end, respectively. The beam has a Young's modulus $E = 10^9$ newton/meter. The area moment of inertia is $I = 10^{-9}$ $(\text{meter})^4$, which corresponds to 1 cm square cross section. Without the extra masses, the beam has a total mass of 1 kilogram. The torsional spring is rather weak with $k = .1$ newton-meter/radian and the linear spring is very strong at $k_1 = 10^{10}$ newtons/meter.

Two independent control forces f_1 and f_2 are applied to each end of the beam. We have derived a finite-element model for this system with 51 modes. The lowest 8 modes and modal shapes are shown in Fig. 2.

4. DECENTRALIZED CONTROL AND SIMULATION RESULTS

To apply the Decentralized Optimal Control technique, we exclude all but the lowest 6 modes in the design model. This continuous-time design model is further transformed into a discrete-time system using a sampling time of 0.01 sec.

In (3), the weighting matrices Q_T and Q are chosen to be $50 \times I_{12}$, and $R_0 = 10^{-5} \times I_2$. We further assume that at each tip of the beam, there is a displacement sensor and a velocity sensor to produce the information for feedback.

By using the formula in (5), we obtain the decentralized feedback law as follows:

$f_1 = -\ 14.2045 \times$ (displacement at the left tip)

$-\ \ 3.9060 \times$ (velocity at the left tip)

$f_2 = -\ 14.2045 \times$ (displacement at the right tip)

$-\ \ 3.9060 \times$ (velocity at the right tip).

Then we apply this decentralized feedback to the 12-th order design model; the resulting closed-loop system is stable with the following eigenvalues:

$0.7625 \pm 0.1933i$

$-0.4212 \pm 0.7023i$

$-0.5613 \pm 0.5999i$

$0.7925 \pm 0.2883i$

$0.8426 \pm 0.1087i$

$0.9226 \pm 0.0255i$

The largest magnitude of these eigenvalues is 0.9230.

Next we perform the time-domain simulation. For this regulation problem, we simulate the time response of the controlled beam starting from the initial bending condition shown by Fig. 3.

A 3-D view of the beam response is shown in Fig. 4. Notice that the vertical axis is not in the same scale as the horizontal axis. The smooth time responses of both tips are shown in Fig. 5.

5. DISCUSSION AND CONCLUSION

In this paper, we have successfully applied the Decentralized Optimal Control Theory [1], [2] to a flexible beam problem. The main features of our approach are the following:

(i) Decentralized control simplifies the structure of feedback controllers significantly.

(ii) Yanchevsky's algorithm produces the controllers in a straightforward manner. The user only needs to specify the weighting matrices Q_T, Q and R_0. In contrast, other methods require trial and error, and often fail to produce a stabilizing controller.

(iii) Yanchevsky's algorithm can produce simple controllers (with few measurements) based on high-order design models. In contrast, other methods usually require low-order design models (excluding high frequency modes) in order to produce simple controllers.

We plan to pursue our future research in the following directions:

(a) Efficient Computational Algorithms

In the above-mentioned design example, we have used a model with six modes. The decentralized feedback gains can be computed rather easily. However, for the design of large scale systems with thousands of modes, we expect to encounter various numerical difficulties and the problem of excessive computational time. In parallel with the usual full-state feedback case, we shall explore the possibility of finding the steady-state feedback matrix by directly solving an algebraic Riccati equation in order to reduce the computational requirement.

(b) Enhancement of Stability Robustness

The design of the controllers is based on a reduced-order model with known parameters. These parameters may differ from the actual parameters of the system. Furthermore, we have excluded the high-frequency dynamics from our design model. These two sources of error may cause the closed-loop system to be unstable. We will study the severeness of this problem in our design. We will also explore ways to enhance the stability robustness to parameter variations and to model order reduction [4], [5].

6. ACKNOWLEDGEMENTS

This research was funded by the Engineering Research Program, Mechanical Engineering Department, Lawrence Livermore National Laboratory (LLNL). The finite element model used in the example was developed by Stan Bumpus of LLNL.

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract No. W-7405-ENG-48.

7. REFERENCES

1. A. E. Yanchevsky and V. J. Hirvonen, "Optimization of Feedback Systems with Constrained Information Flow," Int. J. of Systems Science, Vol. 12, No. 12, 1981, pp. 1459-1468.

2. A. E. Yanchevsky, S. H. Wang and T. C. Hsia, "Decentralized Optimal P, PI and PID Controllers Design for Multivariable Industrial Plants," 19th Annual Asilomar Conference on Circuits, Systems and Computers, Nov. 6-8, 1985, Pacific Grove, CA.

3. D. T. Gavel and H. H. Woo, "Control of a Flexible Beam," Lawrence Livermore National Laboratory, Report No. UCID-20339, January 1985.

4. N. Sundararajan, S. M. Joshi, and E. S. Armstrong, "Robust Controller Synthesis for A Large Flexible Space Antenna," Proceedings of 23rd Conference on Decision and Control, Dec., 1984, pp. 202-208.

5. G. J. Kissel and D. R. Hegg, "Stability Enhancement for Control of Flexible Space Structures," IEEE Control Systems Magazine, Vol. 6, No. 3, June 1986, pp. 19-26.

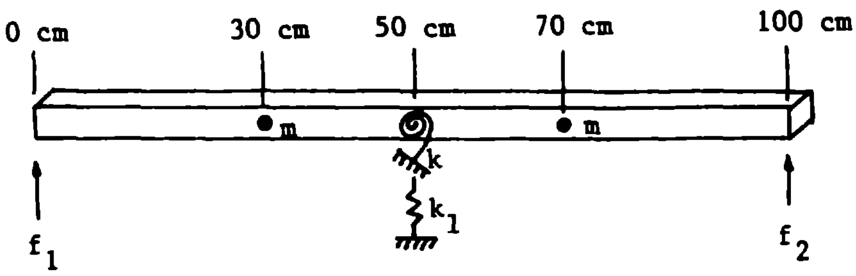

Fig. 1 Beam Configration

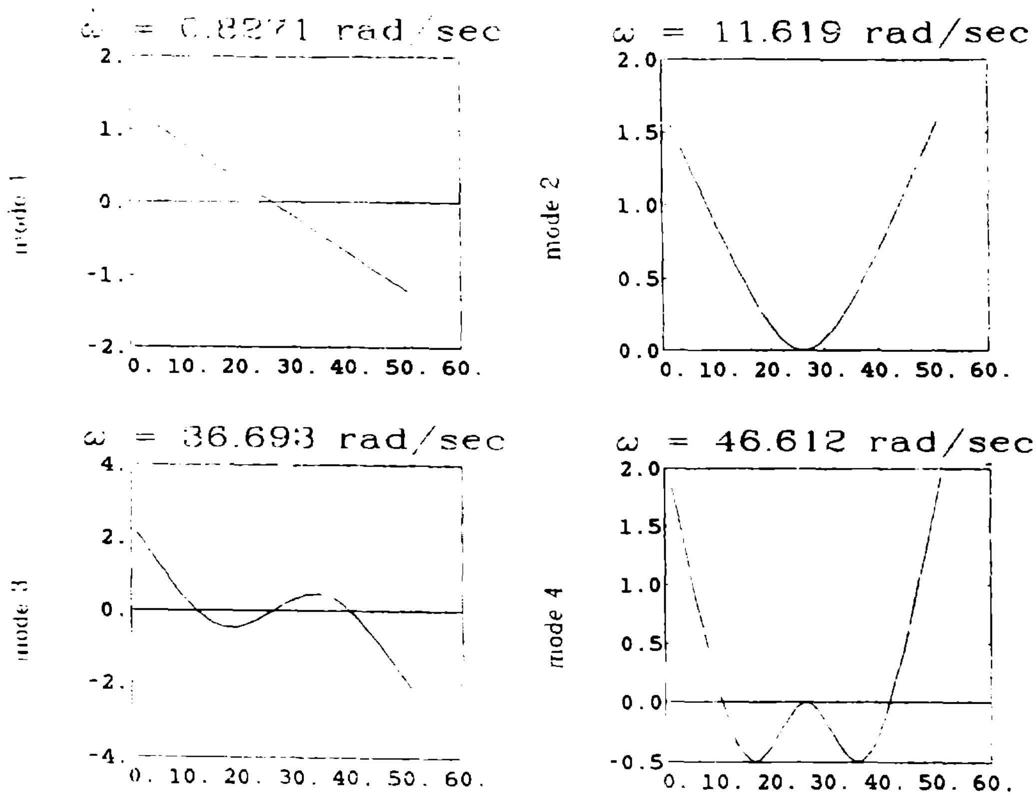

Fig. 2 Beam Vibration Modes (To Be Continued)

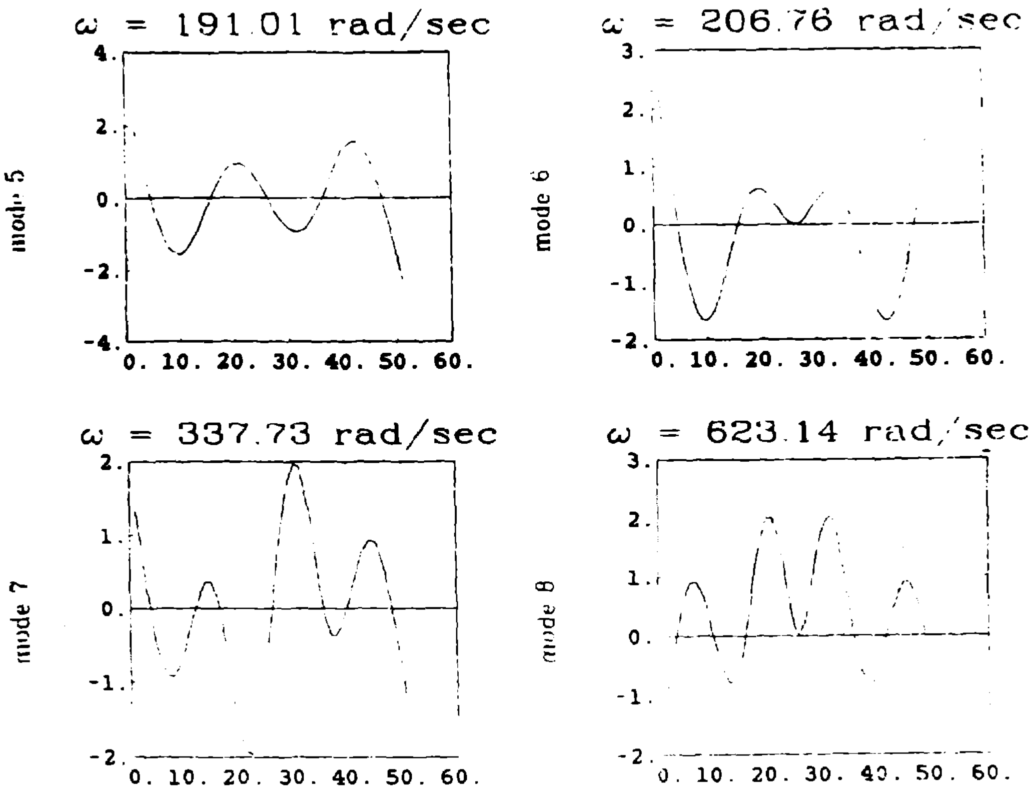

Fig. 2 Beam Vibration Modes (Continued)

Fig. 3 Initial Beam Deformation

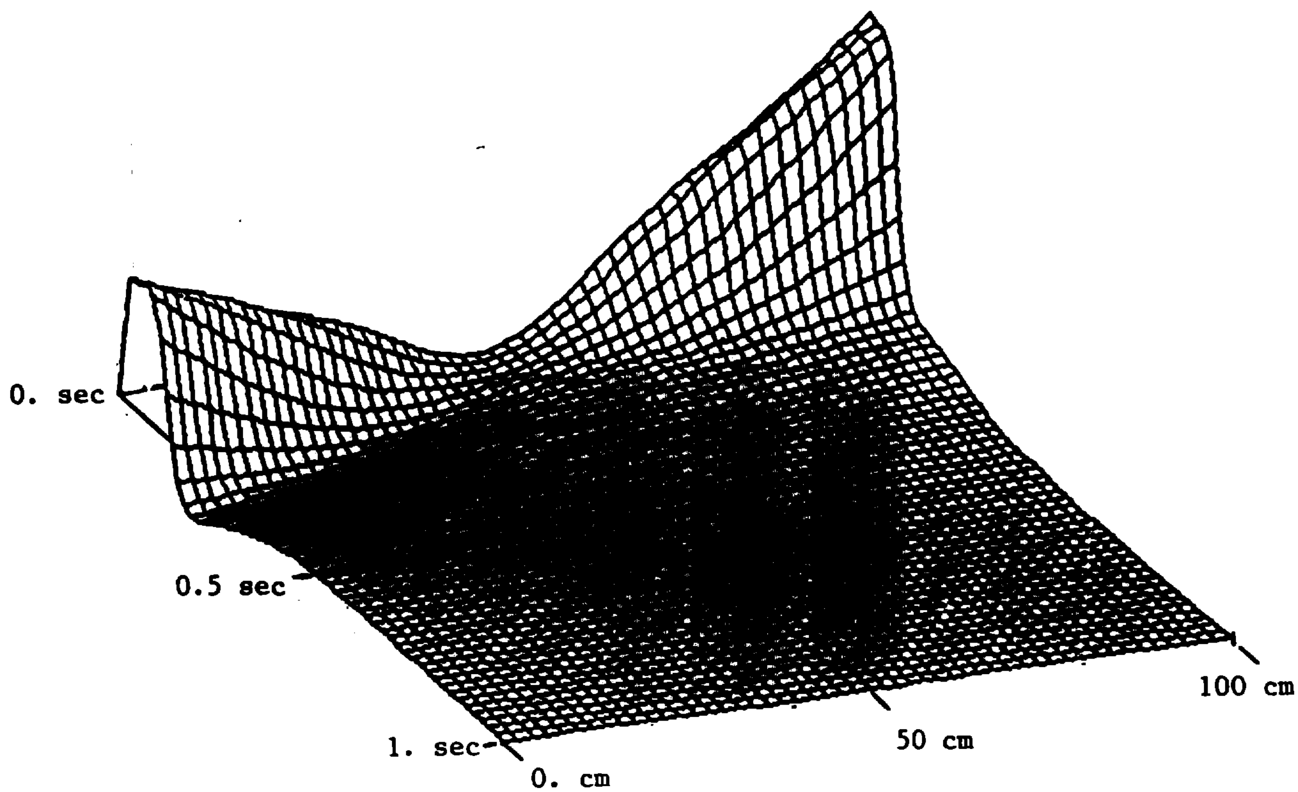

Fig. 4 3D View of the Beam Response

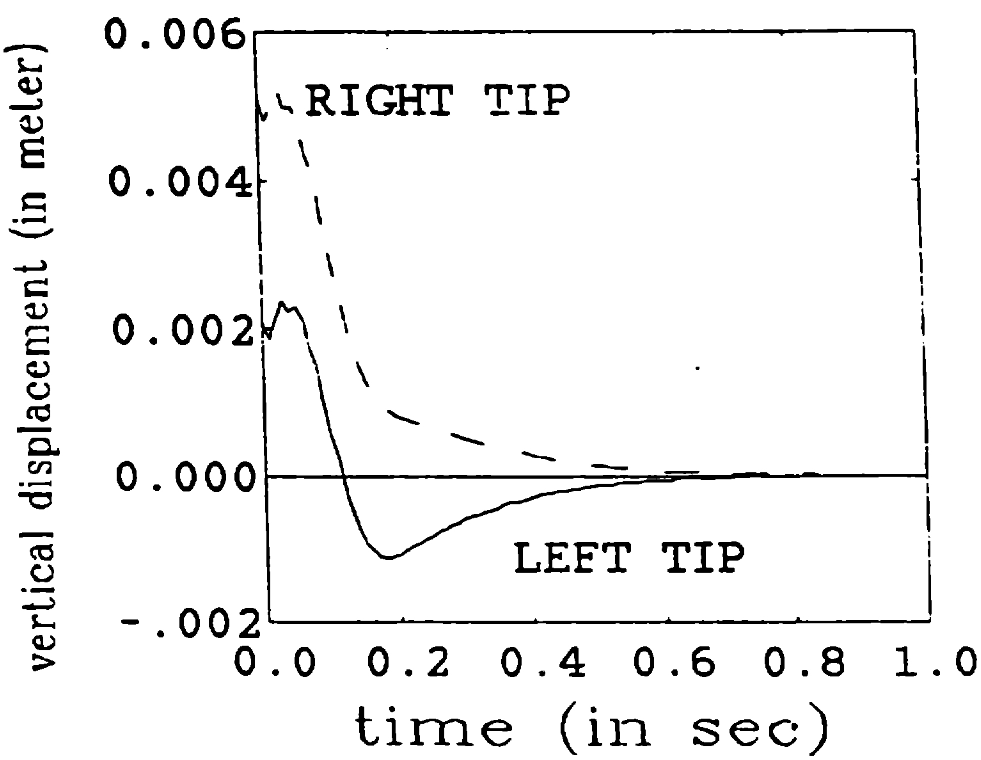

Fig. 5 Beam Response at Tips